Quadrature detection

SmarAct | metrology

INTRODUCTION

A Michelson interferometer, probably the most popular interferometer architecture, is schematically drawn in Figure 1.

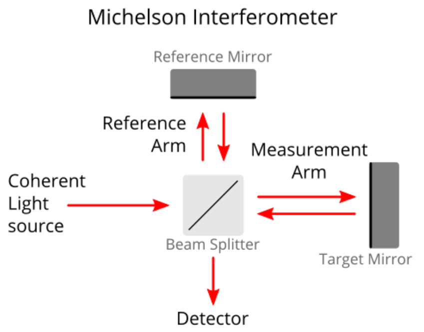

Figure 1. *Basic setup of a Michelson interferometer.*

A coherent light source (laser) is split at a beam splitter into a reference and a measurement arm. Both partial beams are reflected at a reference and target mirror, respectively, and recombine at the beam splitter, forming interferences. The interference signal $I(t)$ measured at the detector can be written as:

$$I(t) = \cos\left(\frac{4\pi}{\lambda} \cdot x(t)\right) \tag{1}$$

where λ is the laser wavelength and $x(t)$ is the length difference between reference and measurement arms.

WAVELENGTH MODULATION

A wavelength modulation of the laser source, by electronically modulating the injection current of appropriate laser diodes for instance[1], can be expressed as:

$$\lambda = \lambda(t) = \lambda_0 + \delta\lambda(t) = \lambda_0 + \delta\lambda \cdot \sin(\omega t)\,, \tag{2}$$

where λ_0 is the carrier wavelength and $\omega/2\pi$ is the modulation frequency.
For practical reasons, $\delta\lambda \ll \lambda_0$ and Equation 2 can be rewritten such that:

$$\frac{1}{\lambda(t)} = \frac{1}{\lambda_0 + \delta\lambda(t)} \approx \frac{1}{\lambda_0} \cdot \left(1 - \frac{\delta\lambda \cdot \sin(\omega t)}{\lambda_0}\right)\,. \tag{3}$$

Equation 1 becomes:

$$\begin{aligned} I(t) &\approx \cos\left(\frac{4\pi}{\lambda_0} \cdot x(t) - \frac{4\pi}{\lambda_0^2} \cdot \delta\lambda \cdot x(t) \cdot \sin(\omega t)\right) \\ &\approx \cos\left(\alpha(t) + z(t) \cdot \sin(\omega t)\right)\,, \end{aligned} \tag{4}$$

[1] For the **PICO**SCALE Interferometer, $\lambda_0 = 1550$ nm, $\delta\lambda < 15$ pm, and $\omega/2\pi = 30$ MHz

where $\alpha(t) = \frac{4\pi}{\lambda_0} \cdot x(t)$ is a phase offset which is independent from the modulation frequency and $z(t) = -\frac{4\pi}{\lambda_0^2} \cdot \delta\lambda \cdot x(t)$ is the modulation depth.

QUADRATURE DETECTION

Equation 4 can be expanded in terms of Bessel functions using the Jacobi-Anger expansion, which yields:

$$\begin{aligned} I(t) \propto\ & S_\omega \cdot \cos(\omega t) + S_{2\omega} \cdot \sin(2\omega t) \\ & + S_{3\omega} \cdot \cos(3\omega t) + O\left[\sin(4\omega t)\right]\,, \end{aligned} \tag{5}$$

with

$$\begin{aligned} S_\omega(t) &\propto J_1(z(t)) \cdot \sin(\alpha(t)) \\ S_{2\omega}(t) &\propto J_2(z(t)) \cdot \cos(\alpha(t)) \\ S_{3\omega}(t) &\propto J_3(z(t)) \cdot \sin(\alpha(t)) \end{aligned} \tag{6}$$

where J_n are the n-th Bessel functions of the first kind.

By lock-in filtering $I(t)$ at frequencies $n \cdot \omega$ it is possible to obtain the coefficients $S_{n\in\{1,2,3,\ldots\}\cdot\omega}$. It is clear that consecutive orders differ in phase by 90°, or said to be **in quadrature**, as for the pairs $(S_\omega;S_{2\omega})$ or $(S_{2\omega};S_{3\omega})$.
When signals in quadrature are plotted against each other, it forms a Lissajous graph, see Figure 2.

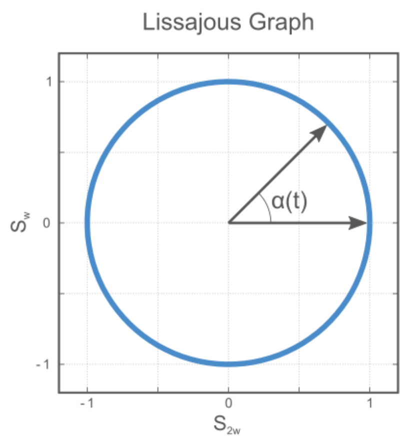

Figure 2. *The pair (S_ω;$S_{2\omega}$), normalized to unity, plotted in a Lissajous graph. The phase $\alpha(t)$ is directly proportional to the displacement $x(t)$.*

When the differential arm length changes, for instance when the target mirror moves, it results in the vector representation of the pair in quadrature moving along a circle. The phase $\alpha(t)$ gives the fraction of one interferometer fringe and by counting full revolutions, macroscopic motions are tracked. Additionally, the direction of motion is revealed. If the distance increases, the vector rotates clockwise, if the distance decreases, the vector rotates counter-clockwise.

In summary, quadrature detection allows to track macroscopic motions with microscopic resolution - including the direction.

For more information on the technique, there exist many scientific publications, for example [1, 2].

REFERENCES

[1] Osami Sasaki. Sinusoidal phase modulating laser diode interferometer with feedback control system to eliminate external disturbance. *Optical Engineering*, 29(12):1511, 1990.

[2] Takamasa Suzuki, Mineki Matsuda, Osami Sasaki, and Takeo Maruyama. Laser-diode interferometer with a photothermal modulation. *Applied Optics*, 38(34):7069, dec 1999.

Contact

Germany

SmarAct Metrology GmbH & Co. KG

Rohdenweg 4
D-26135 Oldenburg
Germany

T: +49 (0) 441 - 800879-0
Email: metrology@smaract.com
www.smaract.com

France

SmarAct GmbH

Schuette-Lanz-Strasse 9
26135 Oldenburg
Germany

T: +49 441 - 800 879 956
Email: info-fr@smaract.com
www.smaract.com

USA

SmarAct Inc.

2140 Shattuck Ave. Suite 302
Berkeley, CA 94704
United States of America

T: +1 415 - 766 9006
Email: info-us@smaract.com
www.smaract.com

China

Dynasense Photonics

6 Taiping Street
Xi Cheng District,
Beijing, China

T: +86 10 - 835 038 53
Email: info@dyna-sense.com
www.dyna-sense.com

Natsu Precision Tech

Room 515, Floor 5, Building 7,
No.18 East Qinghe Anning
Zhuang Road,
Haidian District
Beijing, China

T: +86 18 - 616 715 058
Email: chenye@nano-stage.com
www.nano-stage.com

Shanghai Kingway Optech Co.Ltd

Room 1212, T1 Building
Zhonggeng Global Creative Center
Lane 166, Yuhong Road
Minhang District
Shanghai, China

Tel: +86 21 - 548 469 66
Email: sales@kingway-optech.com
www.kingway-optech.com

Japan

Physix Technology Inc.

Ichikawa-Business-Plaza
4-2-5 Minami-yawata,
Ichikawa-shi
272-0023 Chiba
Japan

T/F: +81 47 - 370 86 00
Email: info-jp@smaract.com
www.physix-tech.com

South Korea

SEUM Tronics

1109, 1, Gasan digital 1-ro
Geumcheon-gu
Seoul, 08594,
Korea

T: +82 2 - 868 10 02
Email: info-kr@smaract.com
www.seumtronics.com

Israel

Optics & Motion Ltd.

P.O.Box 6172
46150 Herzeliya
Israel

T: +972 9 - 950 60 74
Email: info-il@smaract.com
www.opticsmotion.com

SmarAct Metrology GmbH & Co. KG develops sophisticated equipment to serve high accuracy positioning and metrology applications in research and industry within fields such as optics, semiconductors and life sciences. Our broad product portfolio – from miniaturized interferometers and optical encoders for displacement measurements to powerful electrical nanoprobers for the characterization of smallest semiconductor technology nodes – is completed by turnkey scanning microscopes which can be used in vacuum, cryogenic or other harsh environments.
We maintain the complete production in house for a high level of customization so that we can always provide you the optimal individual or OEM solution. We also offer feasibility studies, measurement services and comprehensive support to accompany you along your projects.

Headquarters

SmarAct GmbH

Schuette-Lanz-Strasse 9
26135 Oldenburg
Germany

T: +49 441 - 800 879 0
Email: info-de@smaract.com
www.smaract.com

USA

SmarAct Inc.

2140 Shattuck Ave. Suite 302
Berkeley, CA 94704
United States of America

T: +1 415 - 766 9006
Email: info-us@smaract.com
www.smaract.com