METIRIO Analog Encoder Resolution

Abstract
METIRIO is an optical encoder that detects the relative displacement against a linear or rotary measuring scale and outputs analog incremental signals. In this user guide we show how the position displacement can basically be determined from the analog output signals and how the resolution depends on the electrical noise.

1. TERMS AND DEFINITIONS

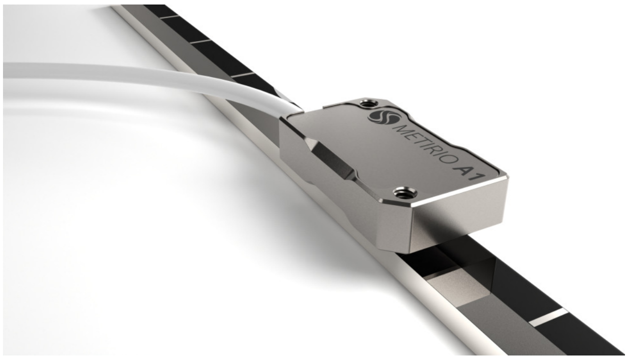

***Figure 1. METIRIO** A1 Readhead with a measuring scale*

When generally talking about measurements, especially displacement measurements, many terms are used to characterize *how good* a measurement is.
In order to avoid confusion we would first like to define the most important terms based on official standards and then focus on the **resolution** as the main characteristic of an analog measuring device. For deeper understanding please refer to:

- JGCM 100:2008 Guide to the expression of uncertainty in measurement (GUM)
- DIN ISO 5275-1 Accuracy (trueness and precision) of measurement results
- DIN ISO 230-2 Test code for machine tools - Determination of accuracy and repeatability of positioning of numerically controlled axes

1.1 Displacement

Depending on the mechanical setup either the readhead or the scale can relatively move in a certain measuring direction. The relative position change between the scale and the readhead along the measuring axis is called **displacement**.

1.2 Resolution

The resolution of an encoder is defined as the least possible displacement that can still be discerned by the evaluation electronics.

> Resolution = least detectable displacement

Among other parameters, the resolution of an analog encoder mainly depends on the following influences:

- The graduation period of the scale,
- the integrated noise of the encoder output signals.

In this user guide we will focus on the **noise**, since this is the only influence parameter related directly to the encoder readhead.

1.3 Further terms

The resolution is the most important property of an analog encoder. Among the other terms mentioned below, only resolution can be mentioned as a property of the read head, since the others depend on the entire sensor system comprising the readhad, a scale and a mechanical assembly. Taking the following definitions into account, this will become more clear.

- **Accuracy:** Accuracy is the ability of a motion system to achieve a commanded position. According to ISO 5275-1 it is defined as the degree of agreement between a test result and an accepted reference value. From this definition it becomes clear why accuracy cannot be a characteristic of the encoder alone, because the position is calculated from the analog voltage signals by a motion controller after digitization.
- **Trueness or correctness:** The trueness is the degree of agreement between the average value obtained from a large series of measurement results and an accepted reference value. It is referred to as *accuracy of the mean*.
- **Precision:** The precision is the degree of agreement between independent test results from a series of measurements with the same system. The precision is not referred to any reference value.

- **Repeatability:** The repeatability is the precision under repeatability conditions, i.e. under conditions where the independent measurement results are obtained with the same method, in the same laboratory by the same operator, equipment etc.

- **Uncertainty:** The uncertainty is used in the GUM as a generic term for both accuracy and precision.

- **Standard deviation:** According to the GUM the uncertainty of a measurement is expressed in terms of standard deviation. For a series of n repetitions of a measurement where x_i is the result of a certain measurement and $\overline{x}$ the average value of the series, the standard deviation is given as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$$

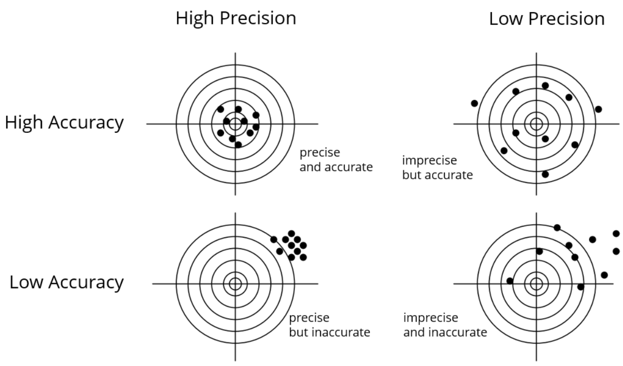

Figure 2. *Illustration of precision and accuracy*

Although these definitions are quite technical, they can be illustrated in a simple picture. According to the definitions the terms **accuracy and trueness** are often used synonymously. Also the terms **precision and repeatability** are often used synonymously, taking into account that precision describes the stricter interpretation of the term, because it is independent of the measurement conditions.
These two characteristics *precision* and *accuracy* are properties of the entire motion system. The resolution of the encoder is one factor that influences these two properties, but not the only one. They further depend on the mechanics and the motion controller. Therefore we will focus on the resolution in the following discussions.

2. ELECTRONIC OPERATING PRINCIPLE

2.1 Sinusoidal output signals

Without going to deep into the optical and electronic details, all optical encoders have the following basic working principle in common. A scale is illuminated by the readhead and the transmitted or reflected light pattern is detected by the readhead via light detecting units, for example an array of photo diodes. This light pattern is translated into a photo current and then via amplifiers into an output voltage.
Depending on the relative displacement to the scale, the detected light pattern changes and so the voltage does. The voltage can vary between a minimum value and a maximum value. The signal will vary periodically between these two values if the displacement extends over several periods of the scale.
Most analog encoders have a second voltage signal that is phase shifted by $\pi/2$. This is needed to detect the moving direction and is also helpful to calculate the position as we will see later.

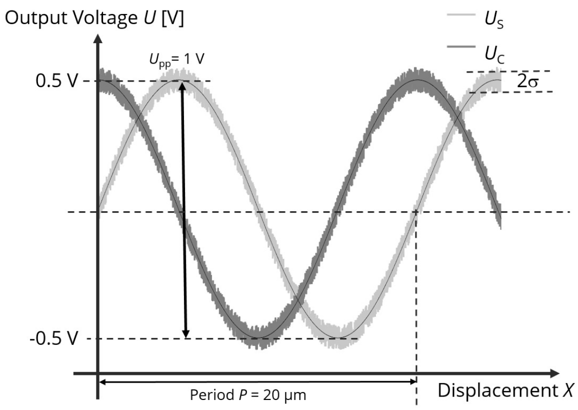

Figure 3. *Illustration of output signals with exaggerated noise*

We are used to looking at electrical signals as a function of time, for example when they are displayed with an oscilloscope. It is worth to mention that encoder signals, when displayed as a function of time, can have any shape. If no movement occurs, they will even be DC-signals, because the output does not change for constant displacement.
However, the encoder output signals U_C and U_S are sine shaped, when displayed as a function of the displacement X. For perfect optical adjustment, both signals have the same amplitude U_{PP}, the same period P and a phase shift of $\pi/2$ as illustrated in figure 3. The period equals the scale periodicity.

2.2 Bandwidth

In simplified terms, the bandwidth is the maximum detectable signal frequency. We will see in section 3 how signal frequency is correlated to the movement speed. If the frequency exceeds a certain value, the signal amplitude U_{PP} will start to drop. Usually the range from 0 Hz to the f_{3dB}-frequency is called the encoder **bandwidth**.

2.3 Noise

Every electronic signal is subject to noise. Noise has several sources with different frequency characteristics. For an electronic device the noise is usually characterized by the amplitude spectral density given in

nV/$\sqrt{\text{Hz}}$. However, when performing a voltage measurement, the total RMS noise U_{RMS} in the frequency band used for the measurement is of interest. The RMS noise is the integral over the noise density in a measured frequency band. The data sheet of an analog encoder shall normally contain this value.
The calculations in section 4 are based on the following assumptions regarding the noise.

Assumption 1: The noise amplitude distribution has a Gaussian shape. This is true for white noise. In this case the RMS noise is the standard deviation $U_{RMS} = 1\sigma$. In this case generally the result of a series of voltage measurements can be written as follows:

$$U = \overline{U} \pm \sigma. \quad (1)$$

Assumption 2: Both output channels for U_C and for U_S have the same noise amplitude, i.e. the same σ.

Assumption 3: The RMS noise amplitude σ is independent of the specific value or phase of the signals $U_{S,C}$.

Assumption 4: Despite the equal RMS noise amplitude, there is no cross-correlation between the two signals. The noise of a particular signal is statistically independent of the other signal.

3. POSITION CALCULATION

3.1 Position without noise

The sine shaped output signals (see figure 3) are a function of the displacement X:

$$U_C = \frac{U_{PP}}{2} \cdot \cos\left(\frac{2\pi}{P} \cdot X\right), \qquad U_S = \frac{U_{PP}}{2} \cdot \sin\left(\frac{2\pi}{P} \cdot X + \delta\right), \quad (2)$$

where the additional phase shift is $\delta = 0$ for perfect optical alignment. The phase of each of the signals depends only on the displacement X.
But why are two signals necessary, in order to calculate the displacement?
Because for a given voltage signal neither the phase nor the direction can be obtained without further information. Moreover the phase change is small near the plateaus, while it is maximum in the zero crossings. Obviously sine and cosine are nonlinear functions.
From the two signals, the phase can be calculated with

$$\boxed{\varphi = \arctan\left(\frac{U_S}{U_C}\right)} \quad (3)$$

As illustrated in figure 4 when the signals are plotted against each other, the resulting graph is a Lissajous-curve. For perfect optical alignment this curve will be

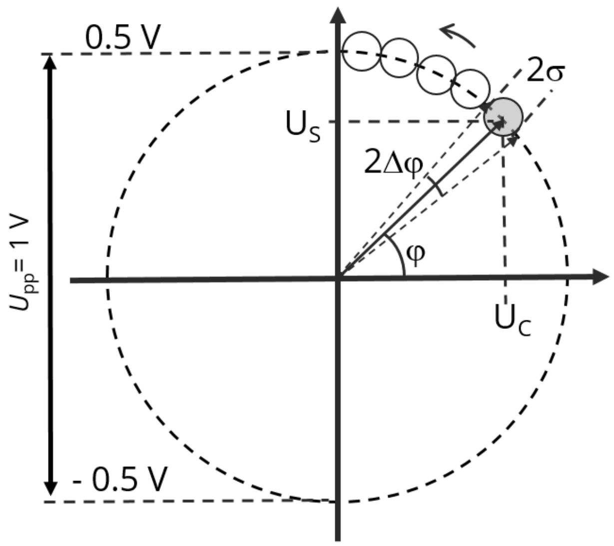

Figure 4. *Lissajous representation of the **METIRIO** output signals for perfect optical alignment and proper gain-settings.*

a perfect circle centered to the coordinate system with the diameter $U_{PP} = 1$ V. Without considering noise, each displacement X_P within one period corresponds exactly to a point on that circle. The phase of this point is given by equation 3.
When the scale moves relative to the readhead, the displacement changes and the point travels on the circle. The **positive movement direction** is defined as the direction for which the point rotates **counter-clockwise**.
Finally the displacement within one period can be calculated from the phase using the following equation:

$$\boxed{X_P = \frac{P \cdot \varphi}{2\pi} = \frac{P}{2\pi} \cdot \arctan\left(\frac{U_S}{U_C}\right)} \quad (4)$$

Why is this type of an encoder called incremental encoder?
Because only relative displacements can be measured. There is no absolute zero position. When moving over several periods, the displacement will increase steadily while the signals vary periodically. The encoder itself does not distinguish between the position within one period and the position within another period. Therefore the controller needs to count the periods of this periodic signals. The number of periods n is obtained by counting, i.e. incrementing, the number of full circumferences. Hence the displacement is

$$X = n \cdot P + X_P. \quad (5)$$

A displacement by one period corresponds to the movement of the point by the full circumference of the circle:

$$\text{displacement } X = P \rightarrow \text{ circumference}: \pi \cdot U_{PP} \quad (6)$$

When the positioning system is actuated backwards, the point will move clockwise and the number n will decrement after each zero-crossing.

3.2 Velocity and bandwidth

What is the maximum displacement velocity v that can be measured with an analog encoder?
As a function of time the analog output signals U_C and U_S will also be sine shaped for a constant velocity. For $v = \text{const.}$ equation 2 can be re-written as:

$$U_C = \frac{U_{PP}}{2} \cdot \cos\left(2\pi \cdot f \cdot t\right), \quad U_S = \frac{U_{PP}}{2} \cdot \sin\left(2\pi \cdot f \cdot t + \delta\right). \tag{7}$$

One period of the signal $T = 1/f$ corresponds to one period P of the scale. For a limited bandwidth, characterized by a maximum frequency f_{3dB}, the maximum velocity is

$$\boxed{v = \frac{P}{T} = P \cdot f_{3dB}} \tag{8}$$

4. POSITION RESOLUTION AND SIGNAL NOISE

Any displacement estimation according to the calculations given in section 3, depends on the measurement of the voltage singals U_C and U_S. The final result is given with an uncertainty

$$X \pm \Delta X \tag{9}$$

since according to equation 1 the voltage measurement is subject to errors due to the noise of the signal. This illustrated in figure 5.

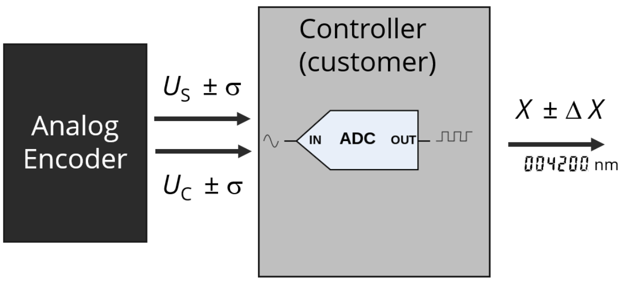

***Figure 5.** Block diagram of signal digitization*

According to figure 4 the phase φ will have an error $\pm\Delta\varphi$, because the point on the lissajous circle is no longer a point, but a disc with diameter 2σ. According to equation 4 the **uncertainty** of the position is given by:

$$\Delta X = \frac{P}{2\pi} \cdot \Delta\varphi, \tag{10}$$

when we assume that the count-number n has no error.

4.1 Resolution vs. Uncertainty

Regarding the definition of the resolution given in section 1 as the least detectable displacement, one might ask how the resolution and uncertainty are related to each other. In fact, there is no difference. Both quantities have the same amount.
With other words: *can the least detectable displacement be smaller than ΔX?* No, it can't. Imagine two series of measurements. Assume that the difference between the two mean values would be less than ΔX. In this case one can not distinguish if a certain measurement value belongs to the first or the second series. Furthermore, if the position is displayed continuously for example as a digital number on a display (see fig. 5) and updated with a certain repetition rate, the number on the display will not be constant, but jitter by ΔX. Only for $X \geqslant \Delta X$, the displacement can be detected by a change of the value shown on the display. The significant decimal place which will not jitter is greater than ΔX.

4.2 Statistical calculation

According to statistics (see Gauss' law of error propagation) the uncertainty of the phase under the conditions from Assumptions 1-4 is given by:

$$\Delta\varphi = \sqrt{\left(\frac{\partial\varphi}{\partial U_S}\right)^2 \cdot \sigma^2 + \left(\frac{\partial\varphi}{\partial U_C}\right)^2 \cdot \sigma^2} \tag{11}$$

Inserting φ from equation 3 and performing the partial derivatives we obtain:

$$\begin{aligned} \Delta\varphi &= \sigma\sqrt{\left(\frac{U_C}{U_S^2 + U_C^2}\right)^2 + \left(\frac{-U_S}{U_S^2 + U_C^2}\right)^2} \\ &= \sigma\sqrt{\frac{U_S^2}{(U_S^2 + U_C^2)^2} + \frac{U_C^2}{(U_S^2 + U_C^2)^2}} \\ &= \sigma\sqrt{\frac{U_S^2 + U_C^2}{(U_S^2 + U_C^2)^2}} \\ &= \sigma\sqrt{\frac{1}{(U_S^2 + U_C^2)}} \end{aligned}$$

Since we assumed a perfect circle (see fig. 4), the relation $U_C^2 + U_S^2 = R^2 = (U_{PP}/2)^2$ holds. Hence it follows:

$$\boxed{\Delta\varphi = 2\sigma/U_{PP}} \tag{12}$$

4.3 Geometrical calculation

The result from equation 12 can also be derived in a geometrical manner. According to figure 4 the following equation holds:

$$\frac{2\Delta\varphi}{2\pi} = \frac{2\sigma}{\pi U_{PP}}, \quad \Delta\varphi = 2\sigma/U_{PP}. \tag{13}$$

4.4 Conclusion: resolution

Inserting the phase error $\Delta\varphi$ into equation 10, we obtain:

$$\boxed{\Delta X_P = \frac{P}{2\pi} \cdot \frac{2\sigma}{U_{PP}} = \frac{P\sigma}{\pi U_{PP}}} \quad (14)$$

5. ADDITIONAL COMMENTS

5.1 Averaging

The integrated RMS noise depends on the bandwidth used for the measurement. Let the measurement frequency be lower than f_{3dB}, in this case the RMS noise within this smaller band will be smaller than the noise at full bandwidth, since the measurement setup averages over several points. Due to the slope of the amplitude spectral density curves, the RMS noise in the band $[0, f_m]$ is proportional to the square root of the integration frequency f_m, if white noise is dominant. From the thumb rule

$$\boxed{\frac{\sigma(f_m)}{\sigma(f_{3dB})} \propto \sqrt{\frac{f_m}{f_{3dB}}}} \quad (15)$$

the RMS noise with a measurement bandwidth of f_m under ideal measurement conditions can be calculated. If for example the maximum velocity can be limited, then the frequency band in the measurement can be smaller than the full bandwidth $f_m < f_{3dB}$.

5.2 Digitization

As mentioned previous in the text, analog encoders output analog voltage signals. All the calculations presented here, have to be performed by the customer's motion controller or some subsequent electronic interface module. Very often, such interfaces convert the two analog signals U_C and U_S into quadrature signals called **A quad B** or **ABZ**. This means, that the sine signals are converted into rectangular signals with a certain interpolation factor k.

This factor k will limit the resolution. Additionally, the electric RMS noise of the ADC within the interface module or the controller can limit the resolution. But in most cases, this noise is kept small. So the digital resolution is limited by k, according to

$$\Delta X_{\text{Dig.}} = P/k, \quad (16)$$

where P is the displacement period according to figure 3, which is given by the stripe size of the scale.

In most cases k will be given in `bit`. Every interface has it's maximum interpolation factor, which is furthermore frequency dependent. So most interface modules will achieve the best resolution only at small velocities while the maximum velocity will lead to a worse resolution.

> The interpolation factor k is frequency dependent.

This means, that the highest interpolation can be set at lowest speed and vice versa. So according to the frequency limits given in the electrical datasheet, the user needs to consider which will be his maximum displacement speed in his application, in order to set the desired interpolation factor k properly.

But how small can ΔX_D be? Can it be smaller than the analog resolution?

Definitely not. Even if one applies a very good interpolation with many bits, so that $\Delta X_{\text{Dig.}} < \Delta X_{\text{Analog}}$, the digital number will jitter as described above. The uncertainty ΔX would be limited to the analog resolution. Therefore the least detectable displacement cannot be decreased by further interpolation. No additional benefit would be achieved. However, if for example due to a small value of k, the digital resolution is greater than the analog one ($\Delta X_{\text{Dig.}} > \Delta X_{\text{Analog}}$), or if the ADC converter adds some additional noise, this worse value will limit the resolution.

The final resolution will always be limited to the worst value in the processing chain.

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