PicoScale signal analysis

Abstract
Signal analysis is a widely used method in technical and scientific disciplines to analyze the properties of time signals and waveforms. Here, we present the functionality of SmarAct's PICO SCALE to detect and calculate the Fourier spectrum of oscillations and dynamic shifts from objects on different length scales with highest time and spacial resolution. This ability makes the PICO SCALE a very powerful tool for challenges in the fields of nanotechnology and microsystem engineering, where signal analysis ranging from the millimeter length scale (10^{-3} m) down to the very small picometer length scale (10^{-12} m) are of interest.

INTRODUCTION
An oscillating and continuous time signal is build up of simple harmonic components with defined amplitudes and frequencies. In the 19th century Joseph Fourier found a simple mathematically expression to describe the composition of periodically functions by simple mathematical expressions. Today, the Fourier analysis is a very helpful tool to analyze data sets in different technical and scientific disciplines like microscopy, laser physics or optics. Combined with digital signal analysis, the discrete Fourier transformation, or DFT, exhibits the Fourier spectrum of signals on every time and length scale. In this note, a short abstract is presented that describes the functionality of SmarAct’s PICO SCALE to calculate the Fourier spectrum of interferometrically obtained data down to the picometer-scale with highest accuracy and precision.

1. FOURIER TRANSFORMATION
Basically, the digital signal analysis samples an analog signal with a defined frame or sample rate f_s and specified number of samples #s (block-size). The obtained waveform consists of discrete values which always means a loss of information and can produce faulty results (e.g. aliasing). Furthermore, the limited time period can lead to spectral leakage in the Fourier domain. To obtain the frequency spectrum of a digitally sampled signal, the discrete Fourier transformation (DFT) is a very helpful tool and shortly discussed in the following section.

1.1 Discrete Fourier transformation (DFT/FFT)
The discrete Fourier transformation (DFT) calculates the Fourier spectrum of a time discrete waveform-signal with the frequency components \( \omega = 2 \pi \cdot f \)

\[
F(t) = \sum C_k e^{i \omega_k t}.
\]  

The complex coefficients \( C_k \) correspond to the amplitudes of the single harmonic functions at \( \omega \). The PICO SCALE control GUI implements a standard LabVIEW function to determine the Fourier spectrum. The Fourier graph in the Stream Monitor panel depicts frequency amplitudes as the root-mean-square values (RMS) in the frequency domain. The frequency separation on the abscissa is given by

\[
\Delta f = f_s/#s.
\]  

and the maximum frequency component \( f_{\text{max}} \) of the spectrum results from

\[
f_{\text{max}} = f_s/2.
\]  

If the block-size is a power of 2 (like 4096, 8192, ...) a special DFT algorithm can be used, called fast Fourier transformation (FFT). The fast Fourier transformation allows a much more efficient calculation of the Fourier spectrum and consequently saves time.
1.2 Spectral leakage
The Fourier transformation tries to calculate the spectrum of an infinitely long waveform. In a realistic case, only a section of the waveform can be used for signal analysis. This artificial restriction leads to a spectral redistribution and a distorted presentation of the Fourier amplitudes. Typically, in signal analysis a window function is applied to the signal to minimize this leakage effect. A Hanning window is used in the PICO SCALE control GUI by default, but also other standard window function can be set as required.

2. SIGNAL ANALYSIS
The range of application of Smaract’s PICO SCALE interferometer concerning signal analysis, ranging from the millimeter down to the picometer scale, is demonstrated by giving three short examples from practice. For each example, the Hanning window function is used for analysis.

2.1 Damped oscillation on the millimeter-scale
In a first example, the damped oscillation of a simple spring-mass (bumper) system in Fig. 1 is detected by the interferometer. Correspondingly, an exponential decrease of the oscillation amplitude (Fig. 2) can be observed in the time domain. The resonance characteristics can be extracted from the Fourier domain by applying a fast Fourier transformation and enables the access to resonance frequency \( f_{\text{res}} = 8 \text{ Hz} \) and damping properties \( Q = 12.5 \) (Q-factor) of the system.

2.2 Nanometer-scale oscillation
In a next step, the spring-mass system is replaced by piezo actuated target mirror. An electronically excited low-frequency oscillation is depicted in Fig. 3 and exhibits an amplitude on the nanometer scale. The desired frame rate is set to 4.88 kHz and the block-size amounts to 8192 samples. The minimal measurement time to fill the block-size is easily be derived by 8192 / 4880 s\(^{-1}\) = 1.68s. In the corresponding Fourier domain, the main oscillation component at 16.6 Hz has an 12 nm (RMS) amplitude, which fits well to the time domain signal. Additionally, the second and third harmonic are recognized due to small non-linearities in the piezo dynamics.

2.3 Picometer-scale oscillation
In Fig. 4 the same signal with a lower voltage amplitude is applied to the piezo-element. In the averaged Fourier spectrum (frame rate: 2.44 kHz / block-size: 8192) the oscillation at 16.6 Hz and at higher harmonics can be recognized with amplitudes down to single picometers. This example demonstrates the ability of SmarAct’s PICO SCALE to measure oscillations with highest accuracy down to the picometer scale.

2.4 Low noise floor
The low noise level in Fig. 4 paves the way towards detections on the picometer scale. The double-log representation reveals a decreasing \(1/f^2\) noise floor which ranges from several tens of picometers for low
frequencies down to single picometers for higher frequencies. This low noise characteristics makes the PICO SCALE an ideal scientific instrument for distance measurements on small length-scales.

3. CONCLUSION

SmarAct’s PICO SCALE is a very powerful and compact scientific instrument for distance measurements and signal analysis on different length scales. Three short examples demonstrate the field of applications ranging from oscillations on the millimeter scale down to very small picometer scale.
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